

**Revision: Calculus (Topic 6)****Coursebook chapters: 12–15****1. Do not use a calculator to answer this question.**

For the function  $f(x) = ax^3 + bx^2 + 4x - 3$ , given that  $f'(2) = 0$  and  $f''(2) = 10$ , find  $a, b \in \mathbb{R}$ .

*(accessible to students on the path to grade 3 or 4) [5 marks]*

**2. Find the exact value of:**

$$\int_{-\pi/3}^{\pi/2} \cos\left(\frac{1}{2}x\right) dx$$

*(accessible to students on the path to grade 3 or 4) [4 marks]*

**3. Do not use a calculator to answer this question.**

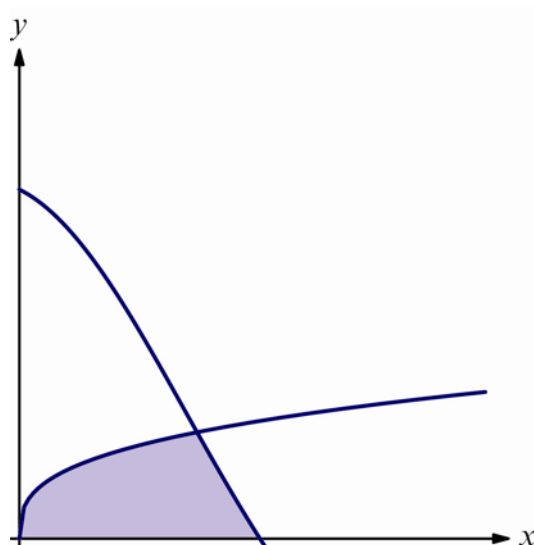
Consider the function:

$$f(x) = \frac{(x-2)(x-6)}{\sqrt{x}}$$

- (a) Show that this can be written in the form  $f(x) = x^a - 8x^b + 12x^c$  giving the values of the real numbers  $a, b$  and  $c$ .
- (b) Find the equation of the normal to  $f(x)$  at the point  $x = 4$ .
- (c) The normal intersects the  $x$ -axis at the point  $P$  and the  $y$ -axis at the point  $Q$ .
  - (i) State the coordinates of  $P$  and  $Q$ .
  - (ii) Give the exact area of the triangle  $POQ$ .

*(accessible to students on the path to grade 3 or 4) [13 marks]*

4. The graph shows curves with equations  $y = \sqrt[3]{x}$  and  $y = 3e^{-x^2} - x$ . Find the area of the shaded region.



*(accessible to students on the path to grade 5 or 6) [6 marks]*

5. **Do not use a calculator to answer this question.**

Consider the curve  $y = 3x^4 - 8x^3 + 6x^2 - 2$ .

- Find the coordinates of all stationary points.
- Classify these stationary points.
- Hence sketch the curve.

*(accessible to students on the path to grade 5 or 6) [12 marks]*

6. By using a suitable substitution or otherwise, find:

$$\int \frac{4x}{x^2 - 9} dx$$

*(accessible to students on the path to grade 5 or 6) [6 marks]*

7. Differentiate the following:

(a)  $y = e^{2x} \tan^2 3x$

(b)  $y = \frac{\ln(1+x^2)}{1+x^2}$

*(accessible to students on the path to grade 5 or 6) [9 marks]*

8. A particle  $P$  moves in a straight line, passing the point  $O$  with speed  $35 \text{ ms}^{-1}$ . At time  $t$  seconds after leaving  $O$  the acceleration  $a \text{ ms}^{-2}$  is given by:

$$a = 6t - 22 \quad 0 \leq t \leq 5$$

- (a) (i) Find an expression for the velocity at time  $t$ .  
 (ii) Find the times at which  $P$  is at rest.  
 (iii) Find the maximum speed of the particle in its 5 second journey.  
 (b) Find the total distance travelled by  $P$ .

*(accessible to students on the path to grade 5 or 6) [11 marks]*

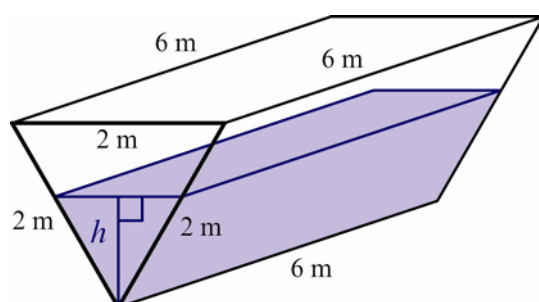
9. (a) Show that:

$$\int \sin^2 x \, dx = \frac{1}{4}(2x - \sin 2x) + c$$

- (b) Hence find the exact value of the volume of the solid formed when  $y = \sqrt{x} \sin x$  for  $0 \leq x \leq \frac{\pi}{2}$  is rotated  $2\pi$  radians about the  $x$ -axis.

*(accessible to students on the path to grade 5 or 6) [12 marks]*

10. A water storage tank is in the shape of a triangular prism, as shown:



Water drips into the tank at a constant rate of  $600 \text{ cm}^3 \text{ s}^{-1}$ .

- (a) Show that the volume of water in the tank (in  $\text{m}^3$ ) when the depth is  $h$  is given by  $V = 2\sqrt{3}h^2$ .  
 (b) Find the rate of increase of the depth of water in the tank at the instant the tank is a quarter full.

*(accessible to students on the path to grade 7) [7 marks]*